Conjecture And Proof

TREVON DOMINGUEZ

The Shape of a Life Basic Books
This text systematically presents the basics of quantum mechanics, emphasizing the role of Lie groups, Lie algebras, and their unitary representations. The mathematical structure of the subject is brought to the fore, intentionally avoiding significant overlap with material from standard physics courses in quantum mechanics and quantum field theory. The level of presentation is attractive to mathematics students looking to learn about both quantum mechanics and representation theory, while also appealing to physics students who would like to know more about the mathematics underlying the subject. This text showcases the numerous differences between typical mathematical and physical treatments of the subject. The latter portions of the book focus on central mathematical objects that occur in the Standard Model of particle physics, underlining the deep and intimate connections between mathematics and the physical world. While an elementary physics course of some kind would be helpful to the reader, no specific background in physics is assumed, making this book accessible to students with a grounding in multivariable calculus and linear algebra. Many exercises are provided to develop the reader's understanding of and facility in quantum-theoretical concepts and calculations.
Uncle Petros and Goldbach's Conjecture Springer Science & Business Media
Looking for a head start in your undergraduate degree in mathematics? Maybe you've already started your degree and feel bewildered by the subject you previously loved? Don't panic! This friendly companion will ease your transition to real mathematical thinking. Working through the book you will develop an arsenal of techniques to help you unlock the meaning of definitions, theorems and proofs, solve problems, and write mathematics effectively. All the major methods of proof - direct method, cases, induction, contradiction and contrapositive - are featured. Concrete examples are used throughout, and you'll get plenty of practice on topics common to many courses such as divisors, Euclidean algorithms, modular arithmetic, equivalence relations, and injectivity and surjectivity of functions. The material has been tested by real students over many years so all the essentials are covered. With over 300 exercises to help you test your progress, you'll soon learn how to think like a mathematician.
Introduction to Mathematical Thinking Oxford University Press on Demand
This introduction to recent developments in algebraic combinatorics illustrates how research in mathematics actually progresses. The author recounts the dramatic search for and discovery of a proof of a counting formula conjectured in the late 1970s: the number of n x n alternating sign matrices, objects that generalize permutation matrices. While it was apparent that the conjecture must be true, the proof was elusive. As a result, researchers became drawn to this problem and made connections to aspects of the invariant theory of Jacobi, Sylvester, Cayley, MacMahon, Schur, and Young; to partitions and plane partitions; to symmetric functions; to hypergeometric and basic hypergeometric series; and, finally, to the six-vertex model of statistical mechanics. This volume is accessible to anyone with a knowledge of linear algebra, and it includes extensive exercises and Mathematica programs to help facilitate personal exploration. Students will learn what mathematicians actually do in an interesting and new area of mathematics, and even researchers

in combinatorics will find something unique within Proofs and Confirmations.

Model Theory and Algebraic Geometry Yale University Press

A New York Times Editors' Pick and Paris Review Staff Pick "A wonderful book." --Patti Smith "I was riveted. Olsson is evocative on curiosity as an appetite of the mind, on the pleasure of glutting oneself on knowledge." --Parul Sehgal, The New York Times An eloquent blend of memoir and biography exploring the Weil siblings, math, and creative inspiration Karen Olsson's stirring and unusual third book, The Weil Conjectures, tells the story of the brilliant Weil siblings—Simone, a philosopher, mystic, and social activist, and André, an influential mathematician—while also recalling the years Olsson spent studying math. As she delves into the lives of these two singular French thinkers, she grapples with their intellectual obsessions and rekindles one of her own. For Olsson, as a math major in college and a writer now, it's the odd detours that lead to discovery, to moments of insight. Thus The Weil Conjectures—an elegant blend of biography and memoir and a meditation on the creative life. Personal, revealing, and approachable, The Weil Conjectures eloquently explores math as it relates to intellectual history, and shows how sometimes the most inexplicable pursuits turn out to be the most rewarding.

Research in Collegiate Mathematics Education III Springer Science & Business Media

Eugène Charles Catalan made his famous conjecture – that 8 and 9 are the only two consecutive perfect powers of natural numbers – in 1844 in a letter to the editor of Crelle's mathematical journal. One hundred and fifty-eight years later, Preda Mihailescu proved it. Catalan's Conjecture presents this spectacular result in a way that is accessible to the advanced undergraduate. The author dissects both Mihailescu's proof and the earlier work it made use of, taking great care to select streamlined and transparent versions of the arguments and to keep the text self-contained. Only in the proof of Thaine's theorem is a little class field theory used; it is hoped that this application will motivate the interested reader to study the theory further. Beautifully clear and concise, this book will appeal not only to specialists in number theory but to anyone interested in seeing the application of the ideas of algebraic number theory to a famous mathematical problem.

How to Think Like a Mathematician Cambridge University Press

"Mathematical thinking is not the same as 'doing math'--unless you are a professional mathematician. For most people, 'doing math' means the application of procedures and symbolic manipulations. Mathematical thinking, in contrast, is what the name reflects, a way of thinking about things in the world that humans have developed over three thousand years. It does not have to be about mathematics at all, which means that many people can benefit from learning this powerful way of thinking, not just mathematicians and scientists."--Back cover.

Proofs from THE BOOK American Mathematical Society

Uncle Petros is a family joke. An ageing recluse, he lives alone in a suburb of Athens, playing chess and tending to his garden. If you didn't know better, you'd surely think he was one of life's failures. But his young nephew suspects otherwise. For Uncle Petros, he discovers, was once a celebrated mathematician, brilliant and foolhardy enough to stake everything on solving a problem that had defied all attempts at proof for nearly three centuries - Goldbach's Conjecture. His quest brings him into contact with some of the century's greatest mathematicians, including the Indian prodigy Ramanujan and the young Alan Turing. But his struggle is lonely and single-minded, and by the end it has apparently destroyed his life. Until that is a final encounter with his nephew opens up to Petros, once more, the deep mysterious beauty of mathematics. Uncle Petros and Goldbach's Conjecture is an inspiring novel of intellectual adventure, proud genius, the exhilaration of pure mathematics - and the rivalry and antagonism which torment those who pursue impossible goals.

Geometrisation of 3-manifolds American Mathematical Soc.

"The Collatz Conjecture" is restricted to positive area of 3n+1 Mapping. Since I applied it to include whole integers (negative and positive integers and zero) of 3n+1 Mapping and also to 3n+b Mappings, I determined many rules, such as "Odd Number's Relation Rule," "Rule to Form Tree Structures" and "Rule to Form Cycles." Also I expanded the Collatz Conjecture to "an+b Mappings" which Thwaites indicated, then I obtained generalized rules. If the problem of the Collatz Conjecture was "10n+6," ordinarily the decimal number system would be used. If the number was odd, the decimal point would be moved to the right side for one digit, and "6" would be placed to fill the space at the unit digit position. I used the radix "a" notation system for the an+b Mapping. If the value of "b" is indicated by a single digit, the value of "b" is used to fill the space at the unit digit position. If it is an even number, half the number will be placed one step down. When the halved number becomes an odd number, it will be replaced with an even number as described above. Therefore, even numbers in the radix "a" notation are lined up in each row. As a result, even numbers are arranged like a belt, making them look like a continent or a long island on a map. The west coast line is made by an arrangement of the most significant non-zero digits of integers. Then, I found out that the west coast line seems like one straight inclined line. I obtained a theory that if a>4, then an+b Mappings are divergent, and if a

Catalan's Conjecture American Mathematical Society

The Budapest semesters in mathematics were initiated with the aim of offering undergraduate courses that convey the tradition of Hungarian mathematics to English-speaking students. This book is an elaborate version of the course on Conjecture and Proof. It gives miniature introductions to various areas of mathematics by presenting some interesting and important, but easily accessible results and methods. The text contains complete proofs of deep results such as the transcendence of e, the Banach-Tarski paradox and the existence of Borel sets of arbitrary (finite) class. One of the purposes is to demonstrate how far one can get from the first principles in just a couple of steps. Prerequisites are kept to a minimum, and any introductory calculus course provides the necessary background for understanding the book. Exercises are included for the benefit of students. However, this book should prove fascinating for any mathematically literate reader.

The Real Fatou Conjecture Springer Science & Business Media

A central concern of number theory is the study of local-to-global principles, which describe the behavior of a global field K in terms of the behavior of various completions of K. This book looks at a specific example of a local-to-global principle: Weil's conjecture on the Tamagawa number of a semisimple algebraic group G over K. In the case where K is the function field of an algebraic curve X, this conjecture counts the number of G-bundles on X (global information) in terms of the reduction of G at the points of X (local information). The goal of this book is to give a conceptual proof of Weil's conjecture, based on the geometry of the moduli stack of G-bundles. Inspired by ideas from algebraic topology, it introduces a theory of factorization homology in the setting l-adic sheaves. Using this theory, Dennis Gaitsgory and Jacob Lurie articulate a different local-to-global principle: a product formula that expresses the cohomology of the moduli stack of G-bundles (a global object) as a tensor product of local factors. Using a version of the Grothendieck-Lefschetz trace formula, Gaitsgory and Lurie show that this product formula implies Weil's conjecture. The proof of the product formula will appear in a sequel volume.

The Ultimate Challenge American Mathematical Soc.

Some years ago a conference on l-adic cohomology in Oberwolfach was held with the aim of reaching an understanding of Deligne's proof of the Weil conjec tures. For the convenience of the speakers the present authors - who were also the organisers of that meeting - prepared short notes containing the central definitions and ideas of the proofs. The unexpected interest for these notes and the various suggestions to publish them encouraged us to work somewhat more on them and fill out the gaps. Our aim was to develop the theory in as self contained and as short a manner as possible. We intended especially to provide a complete introduction to etale and l-adic cohomology theory including the monodromy theory of Lefschetz pencils. Of course, all the central ideas are due to the people who created the theory, especially Grothendieck and Deligne. The main references are the SGA-notes [64-69]. With the kind permission of Professor J. A. Dieudonne we have included in the book that finally resulted his excellent notes on the history of the Weil conjectures, as a second introduction. Our original notes were written in German. However, we finally followed the recommendation made variously to publish the book in English. We had the good fortune that Professor W. Waterhouse and his wife Betty agreed to translate our manuscript. We want to thank them very warmly for their willing involvement in such a tedious task. We are very grateful to the staff of Springer-Verlag for their careful work.

Solved and Unsolved Problems in Number Theory American Mathematical Soc.

The development of mathematical competence -- both by humans as a species over millennia and by individuals over their lifetimes -- is a fascinating aspect of human cognition. This book explores when and why the rudiments of mathematical capability first appeared among human beings, what its fundamental concepts are, and how and why it has grown into the richly branching complex of specialties that it is today. It discusses whether the 'truths' of mathematics are discoveries or inventions, and what prompts the emergence of concepts that appear to be descriptive of nothing in human experience. Also covered is the role of esthetics in mathematics: What exactly are mathematicians seeing when they describe a mathematical entity as 'beautiful'? There is discussion of whether mathematical disability is distinguishable from a general cognitive deficit and whether the potential for mathematical reasoning is best developed through instruction. This volume is unique in the vast range of psychological questions it covers, as revealed in the work habits and products of numerous mathematicians. It provides fascinating reading for researchers and students with an interest in cognition in general and mathematical cognition in particular. Instructors of mathematics will also find the book's insights illuminating.

Conjecture and Proof Springer

Louis de Branges of Purdue University is recognized as the mathematician who proved Bieberbach's conjecture. This book offers insight into the nature of the conjecture, its history and its proof. It is suitable for research mathematicians and analysts.

The Riemann Hypothesis Cambridge University Press

At what point does theory depart the realm of testable hypothesis and come to resemble something like aesthetic speculation, or even theology? The legendary physicist Wolfgang Pauli had a phrase for such ideas: He would describe them as "not even wrong," meaning that they were so incomplete that they could not even be used to make predictions to compare with observations to see whether they were wrong or not. In Peter Woit's view, superstring theory is just such an idea. In Not Even Wrong , he shows that what many physicists call superstring "theory" is not a theory at all. It makes no predictions, even wrong ones, and this very lack of falsifiability is what has allowed the subject to survive and flourish. Not Even Wrong explains why the mathematical conditions for progress in physics are entirely absent from superstring theory today and shows that judgments about scientific statements, which should be based on the logical consistency of argument and experimental evidence, are instead based on the eminence of those claiming to know the truth. In the face of many books from enthusiasts for string theory, this book presents the other side of the story.

Proofs and Confirmations Taylor & Francis

This introduction to the recent exciting developments in the applications of model theory to algebraic geometry, illustrated by E. Hrushovski's model-theoretic proof of the geometric Mordell-Lang Conjecture starts from very basic background and works up to the detailed exposition of Hrushovski's proof, explaining the necessary tools and results from stability theory on the way. The first chapter is an informal introduction to model theory itself, making the book accessible (with a little effort) to readers with no previous knowledge of model theory. The authors have collaborated closely to achieve a coherent and self- contained presentation, whereby the completeness of exposition of the chapters varies according to the existence of other good references, but comments and examples are always provided to give the reader some intuitive understanding of the subject.

The Kepler Conjecture Princeton University Press

2014 Reprint of 1954 American Edition. Full facsimile of the original edition, not reproduced with Optical Recognition Software. This two volume classic comprises two titles: "Patterns of Plausible Inference" and "Induction and Analogy in Mathematics." This is a guide to the practical art of plausible reasoning, particularly in mathematics, but also in every field of human activity. Using mathematics as the example par excellence, Polya shows how even the most rigorous deductive discipline is heavily dependent on techniques of guessing, inductive reasoning, and reasoning by analogy. In solving a problem, the answer must be guessed at before a proof can be given, and guesses are usually made from a knowledge of facts, experience, and hunches. The truly creative mathematician must be a good guesser first and a good prover afterward; many important theorems have been guessed but no proved until much later. In the same way, solutions to problems can be guessed, and a god guesser is much more likely to find a correct solution. This work might have been called "How to Become a Good Guesser."-From the Dust Jacket.

The Weil Conjectures Independently Published

The investigation of three problems, perfect numbers, periodic decimals, and Pythagorean numbers, has given rise to much of elementary number theory. In this book, Daniel Shanks, past editor of Mathematics of Computation, shows how each result leads to further results and conjectures. The outcome is a most exciting and unusual treatment. This edition contains a new chapter presenting research done between 1962 and 1978, emphasizing results that were achieved with the help of computers.

Proofs and Refutations Springer Science & Business Media

Many students have trouble the first time they take a mathematics course in which proofs play a significant role. This new edition of Velleman's successful text will prepare students to make the transition from solving problems to proving theorems by teaching them the techniques needed to read and write proofs. The book begins with the basic concepts of logic and set theory, to familiarize students with the language of mathematics and how it is interpreted. These concepts are used as the basis for a step-by-step breakdown of the most important techniques used in constructing proofs. The author shows how complex proofs are built up from these smaller steps, using detailed 'scratch work' sections to expose the machinery of proofs about the natural numbers, relations, functions, and infinite sets. To give students the opportunity to construct their own proofs, this new edition contains over 200 new exercises, selected solutions, and an introduction to Proof Designer software. No background beyond standard high school mathematics is assumed. This book will be useful to anyone interested in logic and proofs: computer scientists, philosophers, linguists, and of course mathematicians.

Cardinal Arithmetic Cambridge University Press

The Riemann Hypothesis has become the Holy Grail of mathematics in the century and a half since 1859 when Bernhard Riemann, one of the extraordinary mathematical talents of the 19th century, originally posed the problem. While the problem is notoriously difficult, and complicated even to state carefully, it can be loosely formulated as "the number of integers with an even number of prime factors is the same as the number of integers with an odd number of prime factors." The Hypothesis makes a very precise connection between two seemingly unrelated mathematical objects, namely prime numbers and the zeros of analytic functions. If solved, it would give us profound insight into number theory and, in particular, the nature of prime numbers. This book is an introduction to the theory surrounding the Riemann Hypothesis. Part I serves as a compendium of known results and as a primer for the material presented in the 20 original papers contained in Part II. The original papers place the material into historical context and illustrate the motivations for research on and around the Riemann Hypothesis. Several of these papers focus on computation of the zeta function, while others give proofs of the Prime Number Theorem, since the Prime Number Theorem is so closely connected to the Riemann Hypothesis. The text is suitable for a graduate course or seminar or simply as a reference for anyone interested in this extraordinary conjecture.

How to Prove It Macmillan + ORM

Henri Poincaré was one of the greatest mathematicians of the late nineteenth and early twentieth century. He revolutionized the field of topology, which studies properties of geometric configurations that are unchanged by stretching or twisting. The Poincaré conjecture lies at the heart of modern geometry and topology, and even pertains to the possible shape of the universe. The conjecture states that there is only one shape possible for a finite universe in which every loop can be contracted to a single point. Poincaré's conjecture is one of the seven "millennium problems" that bring a one-million-dollar award for a solution. Grigory Perelman, a Russian mathematician, has offered a proof that is likely to win the Fields Medal, the mathematical equivalent of a Nobel prize, in August 2006. He also will almost certainly share a Clay Institute millennium award. In telling the vibrant story of The Poincaré Conjecture, Donal O'Shea makes accessible to general readers for the first time the meaning of the conjecture, and brings alive the field of mathematics and the achievements of generations of mathematicians whose work have led to Perelman's proof of this famous conjecture.